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L0TV: A New Method for Image Restoration in the Presence of Impulse Noise

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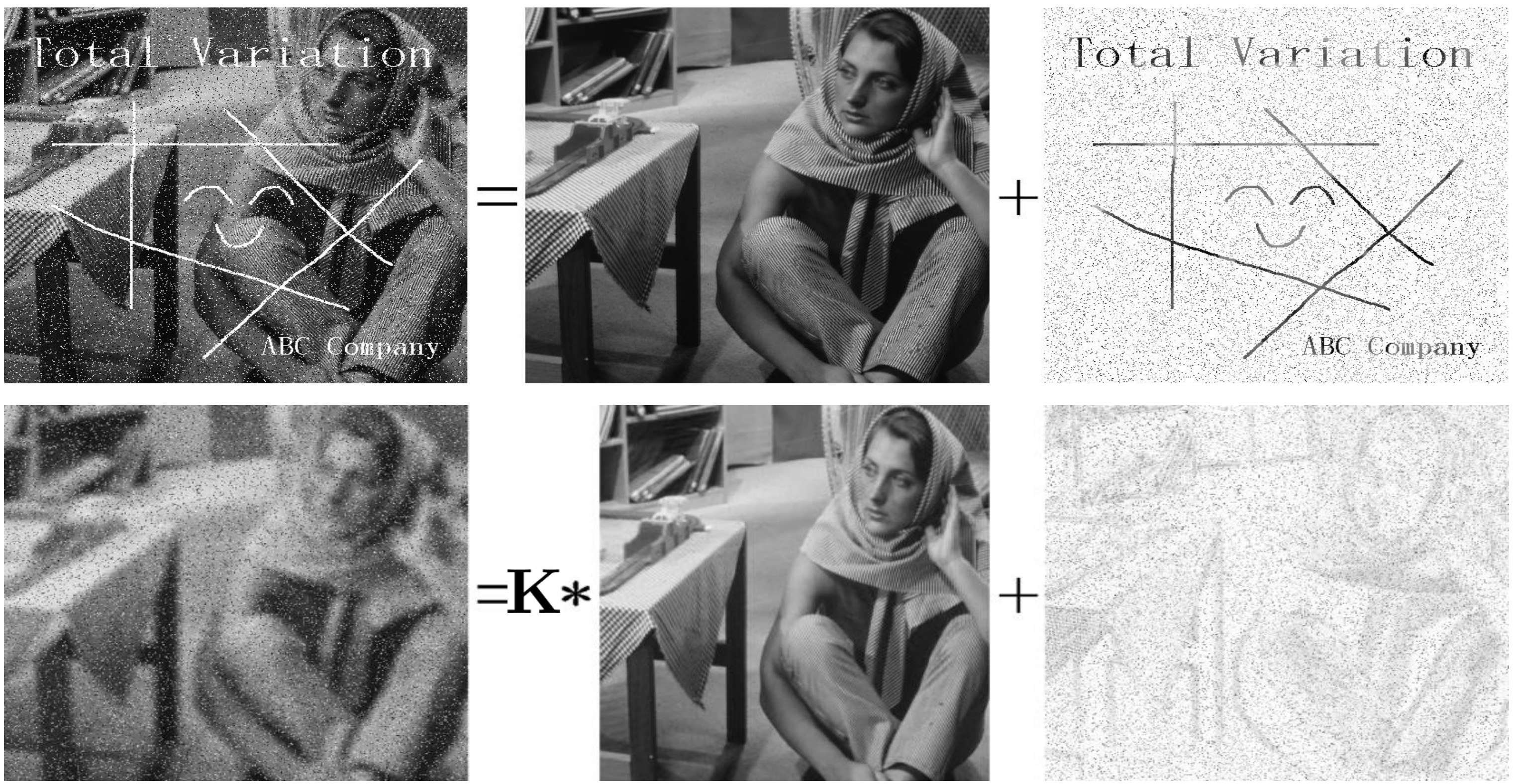
Introduction

- Image restoration problem

$$\mathbf{b} = (\mathbf{K}\mathbf{u} \odot \varepsilon_m) + \varepsilon_a$$



(Generated by our method)

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- Optimization model

$$\min_{\mathbf{u}} \ \ell(\mathbf{K}\mathbf{u}, \mathbf{b}) + \lambda \, \Omega(\mathbf{u})$$

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  - $\Omega(\mathbf{u})$, regularization: **Total Variation (TV)**, Tikhonov-like, Potts, etc.

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  - $\Omega(\mathbf{u})$, regularization: **Total Variation (TV)**, Tikhonov-like, Potts, etc.
  - $\ell(\mathbf{K}\mathbf{u}, \mathbf{b})$, data fidelity: Gaussian, Laplace, etc.

Data Fidelity Models

Data Fidelity Function	Noise	Reference
$\ell_2(\mathbf{Ku}, \mathbf{b}) = \|\mathbf{Ku} - \mathbf{b}\|_2^2$	add. Gaussian	Rudin et al., Physica'92
$\ell_1(\mathbf{Ku}, \mathbf{b}) = \|\mathbf{Ku} - \mathbf{b}\|_1$	add. Laplace	Yang et al., SISC'09
$\ell_\infty(\mathbf{Ku}, \mathbf{b}) = \|\mathbf{Ku} - \mathbf{b}\|_\infty$	add. uniform	Clason, InvProb'12.
$\ell_p(\mathbf{Ku}, \mathbf{b}) = \langle \mathbf{Ku} - \mathbf{b} \odot \log(\mathbf{Ku}), \mathbf{1} \rangle$	add. Poisson	Woo et al., SISC'12
$\ell_g(\mathbf{Ku}, \mathbf{b}) = \langle \log(\mathbf{Ku}) + \mathbf{b} \odot \frac{1}{\mathbf{Ku}}, \mathbf{1} \rangle$	add. Gamma	Aubert et al., SIAP'08
$\ell_{02}(\mathbf{Ku}, \mathbf{b}) = \|\mathbf{Ku} - \mathbf{b} + \mathbf{z}\|_2^2$ $s.t.\ \|\mathbf{z}\|_0 \leq k$	mixed Gaussian impulse	Yan., SIIMS'13
$\ell_0(\mathbf{Ku}, \mathbf{b}) = \|\mathbf{Ku} - \mathbf{b}\|_0$	**add./mult. impulse**	**this paper**

Clean Image

Gaussian Noise

Uniform Noise

Salt-and-Pepper Noise

Contributions

- A new model for impulse noise removal
  - based on L0 norm data fidelity
- A new algorithm for L0 norm optimization
  - based on MPEC (Mathematical Programming with Equilibrium Constraints)

Motivation and Formulation

Motivation and Formulation

- Impulse noise
  - Random-valued, salt-and-pepper
  - Key observation: Impulse noise corrupts a portion of pixels in the image while keeping other pixels unaffected.

- L0TV model

$$\min_{\mathbf{0}\leq\mathbf{u}\leq\mathbf{1}} \|\mathbf{K}\mathbf{u}-\mathbf{b}\|_0 + \lambda TV(\mathbf{u})$$

MPEC Reformulation and Optimization Algorithm

MPEC Reformulation

- **Variational characterization** of the L0 norm

$$\|\mathbf{x}\|_0 = \min_{\mathbf{0}\le\mathbf{v}\le\mathbf{1}} \langle \mathbf{1}, \mathbf{1}-\mathbf{v}\rangle, \ s.t.\ \mathbf{v}\odot|\mathbf{x}| = \mathbf{0}$$

MPEC Reformulation

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- L0TV-MPEC Reformulation

$$\text{L0TV}: \quad \min_{\mathbf{0}\leq\mathbf{u}\leq\mathbf{1}} \|\mathbf{K}\mathbf{u}-\mathbf{b}\|_0 + \lambda TV(\mathbf{u})$$

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$$\Updownarrow$$

$$\text{L0TV-MPEC}: \quad \min_{\mathbf{0}\leq\mathbf{u},\mathbf{v}\leq\mathbf{1}} \langle \mathbf{1}, \mathbf{1}-\mathbf{v}\rangle + \lambda TV(\mathbf{u})$$

$$\text{s.t.} \quad \mathbf{v}\odot|\mathbf{K}\mathbf{u}-\mathbf{b}| = \mathbf{0}$$

MPEC Reformulation

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$$\text{s.t.} \quad \mathbf{v}\odot|\mathbf{K}\mathbf{u}-\mathbf{b}| = \mathbf{0}$$

  - Non-Convex
  - Complementarity constraint: two **non-negative** components, only holds when either of the two components is 0.
  - Practical perspective: desirable solution.

Proximal ADMM for L0TV-MPEC

$$\min_{\mathbf{0}\leq\mathbf{u},\mathbf{v}\leq\mathbf{1}} \quad \langle \mathbf{1}, \mathbf{1}-\mathbf{v}\rangle + \lambda\|\mathbf{x}\|_{p,1}$$

$$s.t. \begin{cases} \nabla\mathbf{u} = \mathbf{x} \\ \mathbf{K}\mathbf{u}-\mathbf{b} = \mathbf{y} \\ \mathbf{v}\odot|\mathbf{y}| = \mathbf{0} \end{cases}$$

Note: $TV(\mathbf{u}) = \|\nabla\mathbf{u}\|_{p,1}$, where $p = 1, 2$

L0TV-PADMM

$$\min_{\mathbf{0}\leq\mathbf{u},\mathbf{v}\leq\mathbf{1}} \quad \langle \mathbf{1}, \mathbf{1}-\mathbf{v}\rangle + \lambda\|\mathbf{x}\|_{p,1}$$

$$s.t. \begin{cases} \nabla\mathbf{u} = \mathbf{x} \\ \mathbf{K}\mathbf{u} - \mathbf{b} = \mathbf{y} \\ \mathbf{v}\odot|\mathbf{y}| = \mathbf{0} \end{cases}$$

Two Features:

- Strongly-convex for each minimization sub-problem
  - unique closed-form solution, the computation in each iteration is insignificant.
- The algorithm is proven to be convergent to a local minimum under mild conditions.

Experimental Results and Future Work

Competing Methods:

- L1TV-SBM, Split Bregman Method: [Goldstein et al., SIIMS 09]
- MFM, Median Filter Method
- TSM, Two Stage Method: [Chan et al., TIP'05]
- L02TV-AOP, Adaptive Outlier Pursuit: [Yan, SIIMS'13]
- L0TV-PDA, Penalty Decomposition Algorithm: [Lu et al., SIOPT' 13]
- L0TV-PADMM, PADMM for solving MPEC of L0TV: [**this paper**]

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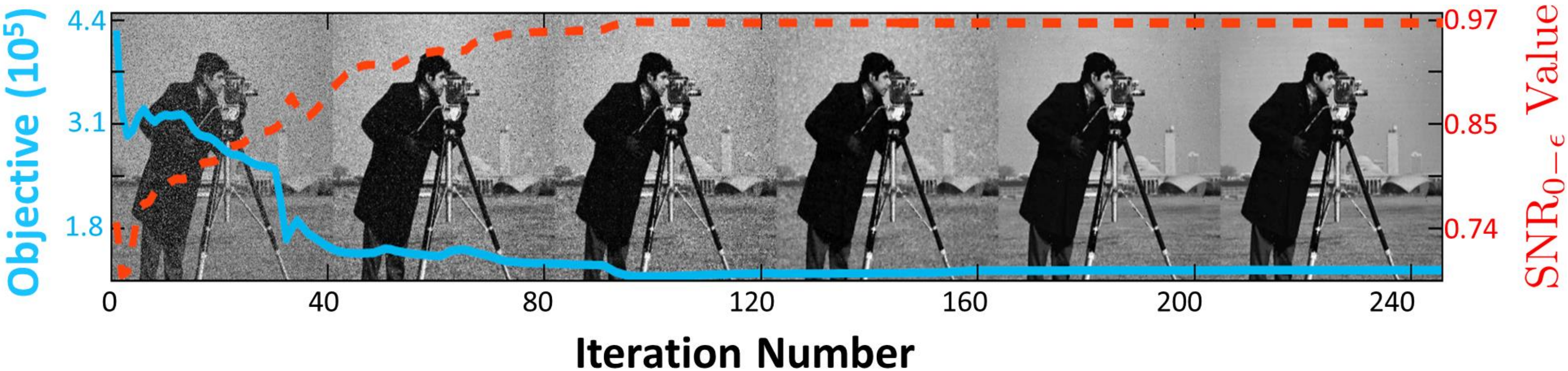
Three ways to measure SNR:

- $\mathrm{SNR}_0(\mathbf{u}, \mathbf{u}^0) \triangleq \frac{n - \|\mathbf{u}^0 - \mathbf{u}\|_0}{n}$
- $\mathrm{SNR}_1(\mathbf{u}, \mathbf{u}^0) \triangleq 10 \log_{10} \frac{\|\mathbf{u}^0 - \bar{\mathbf{u}}\|_1}{\|\mathbf{u} - \bar{\mathbf{u}}\|_1}$
- $\mathrm{SNR}_2(\mathbf{u}, \mathbf{u}^0) \triangleq 10 \log_{10} \frac{\|\mathbf{u}^0 - \bar{\mathbf{u}}\|_2^2}{\|\mathbf{u} - \bar{\mathbf{u}}\|_2^2}$

Convergence Behavior



Observations:

- The objective value decreases and SNR value increase, but they do not do so monotonically.
- Both the objective and SNR values stabilize after 120th iteration.

General Image Denoising Problems

$$\min_{\mathbf{0}\leq\mathbf{u}\leq\mathbf{1}} \|\mathbf{K}\mathbf{u}-\mathbf{b}\|_0 + \lambda TV(\mathbf{u})$$

$$(\text{where } \mathbf{K}=\mathbf{I})$$

General Image Denoising Problems

Img. \ Alg.	$\ell_1 TV$-SBM	MFM	TSM	$\ell_{02} TV$-AOP	$\ell_0 TV$-PDA	$\ell_0 TV$-$PADMM$
Random-Value Impulse Noise						
pirate+10%	0.93/10.06/15.58	**0.97**/14.97/18.50	0.96/13.26/14.26	**0.97**/13.26/17.13	**0.97**/**15.66**/**18.60**	**0.97**/15.46/17.78
pirate+30%	0.88/8.19/12.78	0.91/10.11/12.22	0.85/6.43/8.82	**0.93**/9.36/13.87	**0.93**/**11.46**/**14.88**	**0.93**/11.00/13.12
pirate+50%	0.65/4.69/7.27	0.76/5.53/6.00	0.67/3.16/4.92	0.83/6.95/10.28	0.87/8.64/**11.83**	**0.89**/**8.70**/10.60
pirate+70%	0.42/2.05/2.93	0.53/2.20/1.81	0.46/1.48/2.02	0.55/2.86/3.85	0.62/4.02/5.61	**0.82**/**6.74**/**8.54**
pirate+90%	0.26/0.36/0.12	0.29/0.05/-0.92	0.26/0.21/-0.14	0.28/0.46/0.25	0.31/0.74/0.66	**0.51**/**2.26**/**2.40**
Salt-and-Pepper Impulse Noise						
pirate+10%	0.94/10.18/15.69	0.98/15.29/20.67	0.98/17.54/22.44	0.98/17.53/22.43	**0.99**/19.58/25.95	**0.99**/**19.97**/**26.63**
pirate+30%	0.90/8.66/12.90	0.96/12.58/16.77	0.97/13.80/19.47	0.97/13.76/19.39	**0.98**/14.23/19.98	**0.98**/**14.66**/**20.69**
pirate+50%	0.80/6.43/8.96	0.93/10.19/13.71	**0.96**/11.62/17.05	0.95/11.56/16.94	0.95/11.34/16.57	**0.96**/**11.87**/**17.36**
pirate+70%	0.58/3.21/5.49	0.87/7.99/10.56	**0.92**/9.48/14.10	**0.92**/9.46/14.07	0.89/8.78/13.22	**0.92**/**9.56**/**14.20**
pirate+90%	0.29/1.02/1.78	0.76/5.36/6.67	0.80/6.50/9.64	0.80/6.47/9.58	0.55/3.87/6.39	**0.81**/**6.60**/**9.72**

Observations:

- L0TV-PADMM outperforms all the other methods in most test cases. The performance gap grows larger, as the noise level increases.
- L0TV-PADMM is significantly better than L1TV because we address the l0 norm problem directly instead of approximating it.

General Image Deblurring Problems

$$\min_{\mathbf{0}\leq\mathbf{u}\leq\mathbf{1}} \|\mathbf{K}\mathbf{u}-\mathbf{b}\|_0 + \lambda TV(\mathbf{u})$$

$$(\text{where } \mathbf{K} = \text{Average Kernel})$$

General Image Deblurring Problems

Img. \ Alg.	Corrupted	$\ell_1 TV\text{-}SBM$	TSM	$\ell_{02} TV\text{-}AOP$	$\ell_0 TV\text{-}PDA$	$\ell_0 TV\text{-}PADMM$
Random-Valued Impulse Noise						
cameraman+10%	0.78/5.03/4.83	0.84/7.14/10.46	0.89/7.87/12.20	0.94/10.10/15.92	0.90/8.65/13.00	**0.99/11.14/19.67**
cameraman+30%	0.64/2.39/1.05	0.69/4.54/6.41	0.74/5.26/8.84	0.94/9.99/15.74	0.90/8.41/12.47	**0.97/10.83/18.41**
cameraman+50%	0.50/0.75/-0.96	0.67/3.49/4.23	0.56/3.07/5.31	0.91/8.46/11.52	0.89/8.12/11.92	**0.96/10.45/17.27**
cameraman+70%	0.36/-0.45/-2.36	0.60/2.30/2.40	0.37/1.57/2.50	0.72/3.61/3.51	0.86/7.48/10.75	**0.94/9.75/15.28**
cameraman+90%	0.22/-1.38/-3.40	0.38/1.05/0.98	0.26/0.58/0.70	0.38/0.87/0.75	0.53/2.14/2.71	**0.78/4.94/5.20**
Salt-and-Pepper Impulse Noise						
cameraman+10%	0.78/3.95/2.20	0.87/7.81/11.29	0.95/10.15/16.11	0.95/10.14/16.08	0.91/8.74/13.07	**0.99/11.33/20.85**
cameraman+30%	0.63/0.67/-2.15	0.72/4.77/6.59	0.94/9.97/15.68	0.94/9.98/15.69	0.90/8.56/12.70	**0.99/11.09/20.35**
cameraman+50%	0.47/-1.16/-4.26	0.68/3.62/4.47	0.93/9.58/14.75	0.93/9.53/14.58	0.89/8.30/12.20	**0.98/11.20/19.35**
cameraman+70%	0.32/-2.44/-5.66	0.64/2.67/2.56	0.92/9.09/13.60	0.92/9.03/13.41	0.88/8.00/11.52	**0.97/10.34/17.90**
cameraman+90%	0.17/-3.43/-6.73	0.50/1.24/0.56	0.91/8.79/12.90	0.90/8.59/12.37	0.77/5.67/8.57	**0.93/9.33/14.61**

Observation:

- L0TV-PADMM consistently outperforms all the other methods, by a large margin.

Scratched Image Denoising Problems

Recovered Images

L02TV-AOP

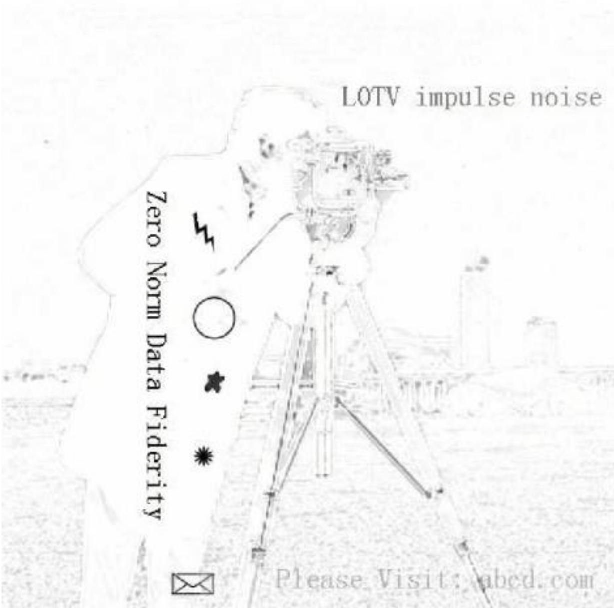
L0TV-PDA

L0TV-PADMM

Observation:

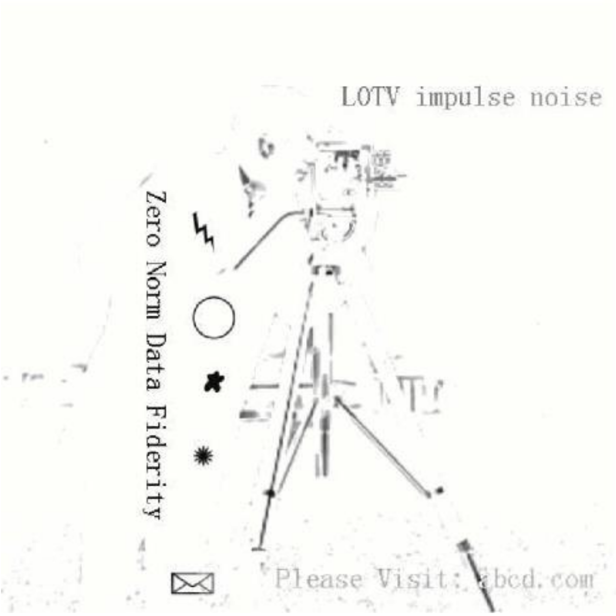
- L0TV-PADMM not only removes the scratches, but it also preserves edges and texture details.

Absolute Residuals



L02TV-AOP

L0TV-PDA



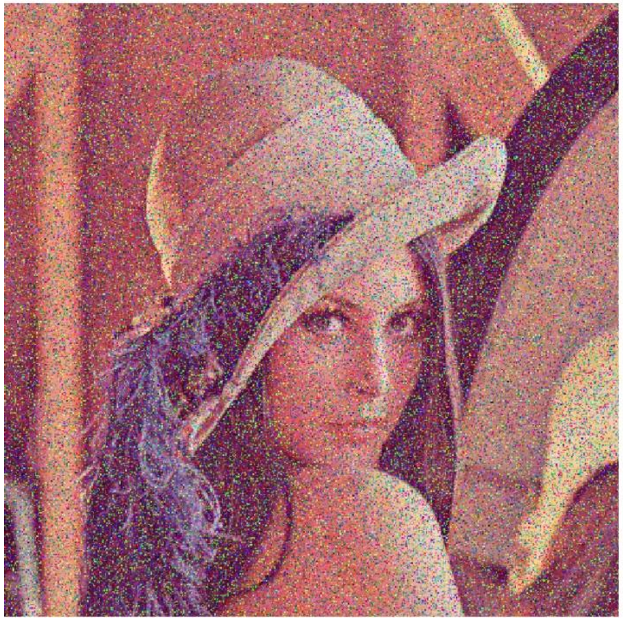
L0TV-PADMM

Observation:

- L0TV-PADMM removes the scratches in the image ONLY.

Color Image Denoising Problems

Clean Image



30% RV Corrupted

Recovered

Observation:

- L0TV-PADMM is practical for impulse noise removal.

Future Work

- Total Variation Prior → Other Prior
  - non-local means, Euler's Elastica, etc.
- MPEC → Other Applications that Optimize L0-Norm
- Cardinality Minimization → Rank Minimization
  - L0 norm of a vector is analogous to the rank of a matrix.

Thank you!

Our code is available online:
http://yuanganzhao.weebly.com/

(Poster #101)